**Barron’s Let’s Review Regents – Algebra II**

# Chapter 4: Radical Expressions and Equations

## 4.1 Simplifying Radicals

**Key Ideas**

A *radical expression* is one that involves a sign (radical sign). Radical expressions are often involved in the solutions to polynomial equations. To work with radicals, you must know how to put them into simplified form and how to combine them.

**Definition of Radicals**

The square root of a number is the thing that must be multiplied by itself to get that number. The symbol for square root is , also called *the radical sign*. An example is because . Even though it is also true that , the symbol means just the positive number that when squared is equal to the number inside the radical sign. A number that is not a perfect square, like 7, still has a square root, although that square root is an irrational number. is between 2 and 3 since and . More precisely, is approximately 2.645751311.

If there is a small number outside the radical sign, it no longer indicates a square root. If there is a small 3 outside the radical sign, it becomes a cube root sign and is equal to the number that must be cubed to become the number under the radical sign. An example is , because . The small number outside the radical sign is called the *index*. When there is no index, it is implied to be a 2. So a radical sign with no index is called the *square root sign*.

**Multiplying Radicals**

Two radical expressions that have the same index can be multiplied by multiplying the numbers inside the radical sign. For example, . This is easily verified for this example since .

**Math Facts**

In general, . This rule also works in reverse for factoring radicals: .

**Simplifying Square Roots**

If the number inside a square root sign has a factor that is a perfect square, the radical can be *simplified*. The can be simplified since one of the factors of 50 is 25, which is a perfect square.

. The 5 in this expression is not an index but a coefficient in front of the radical sign. The multiplication sign between the coefficient and the radical sign is not necessary.

**Example 1**

Simplify the expression .

*Solution*: Since .

It is not always clear whether or not a large number has a factor that is a perfect square. By factoring the number into its *prime factors*, it is possible to group the matching factors into pairs and use the fact that .

Radical expressions that involve variables can also be simplified with this approach.

**Example 2**

Simplify the expression:

*Solution*:

**Adding and Subtracting Radicals**

Radicals can be added or subtracted only if they have the same index and the same number inside the radical sign. They are combined the same way that like terms are combined with polynomials. For example, . Subtraction works the same way: .

If two radical expressions have the same index but different numbers inside the radical sign, you cannot immediately add or subtract them. Sometimes after simplifying the expressions, they will have the same number inside the radical sign and can then be added or subtracted.

**Example 3**

Simplify the terms and combine if possible.

*Solution*: After simplifying:

### Check Your Understanding of Section 4.1

1. Multiple-Choice
2. What is ?  
   **(2) 71**
3. Which of the following is equivalent to ?  
   **(4)**
4. Which of the following is equivalent to ?  
   **(1)**
5. What is ?  
   **(2)**
6. What is ?  
   **(1)**
7. Which of the following is equivalent to ?  
   **(4)**
8. Which of the following is equivalent to   
   **(2)**
9. What is ?  
   **(3) 71**
10. Simplify .  
    **(2)**
11. What is ?  
    **(1) 30**
12. *Show how you arrived at your answers*.
13. Spencer simplifies as   
    . Mia simplifies it as .  
      
    **Mia is correct. There is no distributive property for radicals.  
      
    Radicals can be added or subtracted only if they have the same index and the same number inside the radical sign.  
      
    The square root of a sum is not equal to the sum of the square roots: .**
14. Daniel calculates . Is this correct? Why or why not?  
      
    **No. The square root of a sum is not equal to the sum of the square roots:   
    .  
      
    Radicals can be added or subtracted only if they have the same index and the same number inside the radical sign.**
15. In this right triangle, what is the length of leg simplified in simplest terms?
16. What is
17. What is ?

## 4.2 Imaginary and Complex Numbers

**Key Ideas**

An *imaginary number* is a number that, when multiplied by itself results in a negative number. The most basic imaginary number is since . The number is abbreviated by the symbol *i*. Based on this definition, . When imaginary numbers are combined with real numbers, the result is called a *complex number*. Complex numbers can be added, subtracted, and multiplied with rules similar to polynomial adding, subtracting and multiplying.

**Simplifying Imaginary Numbers**

The square root of any negative number can be represented as an expression involving an . To simplify , split it into .

If the number inside the radical is not a perfect square, like . If the number inside the radical has a factor that is a perfect square, like , the will be written between the coefficient and the radical.  
.

**Adding and Subtracting Imaginary Numbers**

To add or subtract to imaginary numbers like , they are considered to be like terms. So they can be combined as you would do with variables,   
. For subtraction, it would be   
.

**Multiplying Imaginary Numbers**

Multiplying imaginary numbers like is similar to multiplying expressions like . Since would be , is . Since ,   
.

**Example 1**

Multiply .

*Solution*: .

**Powers of**

can be raised to other powers besides 2. When is raised to an integer power, the only possible values are or 1.

|  |  |  |
| --- | --- | --- |
| **Power** |  | **Solution** |
| 0 |  | 1 |
| 1 |  |  |
| 2 |  | -1 |
| 3 |  | -i |
| 4 |  | 1 |

After the power of 3, the powers of cycle between the for values: and . raised to any multiple of 4 will be 1.

To raise to a high power, like , find the multiple of 4 smaller than 47 that is closest to it and rewrite as  
.

**Complex Numbers**

When a real number, like 3, is added to an imaginary number, like , together the sum is written as . A complex number is a number of the form , where and are real numbers.

**Adding and Subtracting Complex Numbers**

Complex numbers are added by adding the real parts and the imaginary parts separately and writing the answer in form.

Subtracting complex numbers requires care as you distribute the -1 through the second expression.

**Multiplying Complex Numbers**

Complex numbers can be multiplied the way binomials are multiplied by using the FOIL shortcut. Anytime an is encountered in a solution, it should be converted to a -1.

To multiply with FOIL:

**Complex Solutions to Quadratic Equations**

If in the process of using the quadratic formula a negative number appears inside the radical, the quadratic equation is said to have two complex solutions.

**Example 4**

Use the quadratic equation to solve   
 with and .

Solution:

Since there is a negative number inside the radical sign, the solutions will be complex numbers.

There are two complex solutions:   
.

**Graphing Complex Numbers on the Complex Plane**

A number like cannot be graphed on the standard number line. Instead, complex numbers are graphed on something called *the complex plane*. The complex plain is like a number line for complex numbers. The complex plane resembles the two-axis coordinate plane. Instead of the axes representing the x-coordinate and the y-coordinate, the axes represent the real part of the complex number and the imaginary part of the complex number.

Below is the complex plane. The number is plotted as a single point in the position (2, 3).

Real Numbers like 2 can be thought of as and are graphed on the real axis of the complex plane. Imaginary numbers like can be thought of as and are graphed on the imaginary axes of the complex plane.

**Math Facts**

The absolute value of a complex number is the distance on the complex plane between that number and the number . The absolute value of a complex number can be calculated with the formula .

**Example 5**

What is ?  
(1) 5 (2) -5 (3) 7 (4) -7

*Solution*:   
So choice (1) is the answer.

### Check Your Understanding of Section 4.2

1. Multiple-Choice
2. What is ?  
   **(1)**
3. What is ?  
   **(4)**
4. What is ?  
   **(1) 21**
5. What is ?  
   **(1) -1**
6. What is ?  
   **(4)**
7. What is ?  
   **(2)**
8. What is ?  
   **(2)**
9. What is ?  
      
   **(1) 34**
10. Solve the quadratic equation   
    .  
    **(1)**
11. What is ?  
    **(3) 13**